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Mutual Dependence between a Bosonic Black Hole and Dark Matter and the Explanation of Asymptotically Flat Galaxy Rotation Curves

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Abstract—The possibility of an equilibrium static state of a collapsed black hole, surrounded by dark matter, makes it possible to understand the existence of flat rotation curves of stars on the periphery of a galaxy. Under the dominant gravity, a Bose–Einstein condensate is the energetically most favourable state of an extremely compressed black hole. It turned out that the longitudinal vector field, as a wave function, adequately describes the observed manifestations of dark matter. Considering as an example a condensate of Z, W, and H bosons of the Standard Model of Elementary Particles (with rest energy of the order of 100 GeV), the dependence of rotation curves of stars on the mass of a black hole at the galaxy center was investigated. With this composition of the black hole of a mass on the order of the solar mass (2×10^{33} g), the dark matter gives the dominant contribution to the gravitational field. In this case, the plateau on the galaxy rotation curve is explicitly expressed. As the black hole mass increases, a contribution to the gravity from the dark matter decreases, while a contribution from the black hole increases. The mass of the black hole at the center of the Milky Way galaxy is seven orders of magnitude greater than the solar mass. The contribution to the gravity from the black hole dominates. Therefore, in our galaxy, the rotation velocity of stars $V(r)$ as a function of radius decreases in proportion to $1/\sqrt{r}$ in accordance with Newton's law.

Keywords: dark matter, black hole, galaxy rotation curve

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INTRODUCTION

Dark matter manifests itself only through the gravitational interaction. Quanta of ordinary matter in flat space are described by vector fields [1]. Let us assume that a wave function of dark matter quanta is also a vector field φ_m. Then it makes sense to find such a vector field in the general theory of relativity (GTR), which reveals itself exclusively in a curved space [2, 3].

LONGITUDINAL VECTOR FIELD

Within the minimal general theory of relativity (field equations of no higher than second order), the Lagrangian L of a vector field φ_m is a scalar S, consisting of a convolution of bilinear combinations of covariant derivatives $\varphi_{i;k}$ and a scalar potential $V\left(\varphi^k\varphi_k\right)$. A bilinear combination of covariant derivatives $\varphi_{i;k}$ is the tensor

$$S_{iklm} = \varphi_{i;k}\varphi_{l;m}. \quad (1)$$

The general form of the Lagrangian L generated by the convolution S of tensor (1) is given by

$$L = a\left(\varphi^m_{;m}\right)^2 + b\varphi^l_{;m}\varphi_l^{;m} + c\varphi^l_{;m}\varphi^m_{;l} - V\left(\varphi_m\varphi^m\right). \quad (2)$$

Here a, b, and c are arbitrary constants. In GTR, the second derivative of a vector is not invariant with respect to an interchange of the order of differentiation:

$$\varphi^l_{;l;m} - \varphi^l_{;m;l} = R_{km}\varphi^k.$$

R_{km} is the Ricci tensor. In a curved space, all three kinetic terms in Lagrangian (2) are equisignificant.

The covariant derivative $\varphi_{i;k}$ can be represented as a sum $\varphi_{i;k} = G_{ik} + F_{ik}$ of the symmetric G_{ik} and antisymmetric F_{ik} tensors:

$$G_{ik} = \frac{1}{2}\left(\varphi_{i;k} + \varphi_{k;i}\right), \quad F_{ik} = \frac{1}{2}\left(\varphi_{i;k} - \varphi_{k;i}\right).$$

The scalar S can be presented as

$$S = a\left(G^k_k\right)^2 + (b+c)\,G^i_kG^k_i + (b-c)\,F^i_kF^k_i.$$

In a flat space $R_{km} = 0$, and in fact only two of the three kinetic terms are independent. As applied to the ordinary matter in a flat space, the gauge invariance allows $a = 0$ to be set. Then the covariant divergence $\varphi^m_{;m}$ becomes an arbitrary function that does not affect the action. In electrodynamics, $\varphi^m_{;m} = 0$ is called the Lorentz gauge ([4], p. 145).

If we set $b = c = 0$, then at $a \neq 0$ we obtain the Lagrangian

$$L = a\left(\varphi^m_{;m}\right)^2 - V\left(\varphi_m\varphi^m\right)$$

of a longitudinal vector field, which, due to the gauge invariance, does not affect the ordinary matter in a flat space. It turns out that in a curved space-time, the longitudinal vector field φ_m adequately describes the observed properties of dark matter [2]. The Euler–Lagrange equation implies the wave equation

$$a\varphi^m_{;m,k} = -V'\varphi_k, \quad V' = \frac{dV}{d\left(\varphi_m\varphi^m\right)}. \tag{3}$$

On a galactic scale, the gravitational interaction is dominant. In the potential expansion

$$V\left(\varphi_m\varphi^m\right) = V_0 + V'(0)\varphi_m\varphi^m + \lambda\left(\varphi_m\varphi^m\right)^2 + \ldots,$$
$$-\frac{V'(0)}{a} = \left(\frac{\mu c}{\hbar}\right)^2 \equiv \left(\frac{1}{\lambda\!\!\!^-}\right)^2$$

the first term V_0 is an addition to the cosmological constant that affects the expansion of the Universe [5]. On the galactic scale, the role of V_0 is negligible. The second term of the expansion $V'(0)\varphi_m\varphi^m$ is the main source of the gravitational interaction; μ is the rest mass of a quantum of the longitudinal vector field. The third term $\lambda\left(\varphi_m\varphi^m\right)^2$ is a correction for interactions of the nongravitational nature (including the elasticity of matter). The coefficient λ in the third term $\lambda\left(\varphi_m\varphi^m\right)^2$ should not be confused with the metric function $\lambda(r) = \ln(-g_{rr})$ in (4) and with the De Broglie wavelength $\lambda\!\!\!^- = \hbar/\mu c$. Restricting ourselves to the term $V'(0)\varphi_m\varphi^m$, we consider the gravitating dark matter as an ideal gas. The mass of a quantum of the longitudinal vector field is denoted μ so as not to be further confused with the mass m of a quantum of the bosonic scalar field of the black hole.

In space-time with a static centrally symmetric Schwarzschild metric [6]

$$ds^2 = g_{ik}dx^i dx^k = e^{\nu(r)}(dx^0)^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2, \tag{4}$$

the energy–momentum tensor $T^k_{\text{dark}\,i}$ of the longitudinal vector field φ_m is written as

$$T^k_{\text{dark}\,i} = \delta^k_i \begin{cases} a\left(\varphi^m_{;m}\right)^2 - V_0' e^\lambda \left(\varphi^r\right)^2, & i = r, \\ a\left(\varphi^m_{;m}\right)^2 + V_0' e^\lambda \left(\varphi^r\right)^2, & i \neq r. \end{cases}$$

The gravitational properties of a longitudinal vector field are described by the Einstein equations

$$R_{ik} - \frac{1}{2}g_{ik}R = \kappa T_{\text{dark}\,ik}.$$

According to the observed rotation curves of galaxies, the de Broglie wavelength $\lambda\!\!\!^- = \hbar/\mu c$ of the longitudinal vector field φ_m is about 15 kpc (see below, Fig. 4). This is many orders of magnitude larger than the size of a black hole. The surface radius of a black hole r_h in our Milky Way galaxy is less than 0.0002 ly [7]. Therefore, the covariant divergence of longitudinal field $\varphi^m_{;m}(r_\text{h})$ is almost indistinguishable from $\varphi^m_{;m}(0)$. In the asymptotic region $r \sim \lambda\!\!\!^- \gg r_\text{h}$, the metric function is $\lambda(r) \ll 1$. Linearized Einstein's equations for metric functions $\nu(r)$, $\lambda(r)$

$$\begin{aligned} \nu' &= \kappa r\left[\left(\frac{1}{\lambda\!\!\!^-}\varphi^r\right)^2 + \left(\varphi^m_{;m}\right)^2\right] + \frac{\lambda}{r}, \\ \lambda' + \frac{\lambda}{r} &= \kappa r\left[\left(\frac{1}{\lambda\!\!\!^-}\varphi^r\right)^2 - \left(\varphi^m_{;m}\right)^2\right] \end{aligned} \tag{5}$$

together with the Klein–Gordon equation for the covariant divergence $\varphi^m_{;m}(r)$ of a longitudinal vector field

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\varphi^m_{;m}}{dr} + \frac{1}{\lambda\!\!\!^{-2}}\varphi^m_{;m} = 0 \tag{6}$$

make it possible to find the dependence on the radius of the speed of a star's rotation around the center $V(r)$ in the asymptotic region $r \gg r_\text{h}$ [8].

The solution to Klein–Gordon equation (6), which is regular at the center, has the following form:

$$\varphi^m_{;m}(r) = \varphi^m_{;m}(0)\frac{\lambda\!\!\!^-}{r}\sin\left(\frac{r}{\lambda\!\!\!^-}\right). \tag{7}$$

From wave Eq. (3), we obtain:

$$\varphi^r(r) = -\varphi^m_{;m}(0)\frac{\lambda\!\!\!^{-3}}{r^2}\left[\sin\left(\frac{r}{\lambda\!\!\!^-}\right) - \frac{r}{\lambda\!\!\!^-}\cos\left(\frac{r}{\lambda\!\!\!^-}\right)\right]. \tag{8}$$

By substituting (7) and (8) into linearized Einstein's equations (5) we find:

$$\begin{aligned} r\frac{d\nu}{dr} &= \kappa\left(\varphi^m_{;m}(0)\right)^2\lambda\!\!\!^{-2} \\ &\times\left[1 - \frac{\lambda\!\!\!^-}{r}\sin\left(\frac{2}{\lambda\!\!\!^-}r\right) + \left(\frac{\lambda\!\!\!^-}{r}\right)^2\sin^2\left(\frac{r}{\lambda\!\!\!^-}\right)\right] + \lambda, \end{aligned} \tag{9}$$

$$\lambda(r) = \kappa\left(\varphi^m_{;m}(0)\right)^2\lambda\!\!\!^{-2}\left[\frac{\lambda\!\!\!^-}{2r}\sin\left(\frac{2}{\lambda\!\!\!^-}r\right) - \left(\frac{\lambda\!\!\!^-}{r}\right)^2\sin^2\left(\frac{r}{\lambda\!\!\!^-}\right)\right]. \tag{10}$$

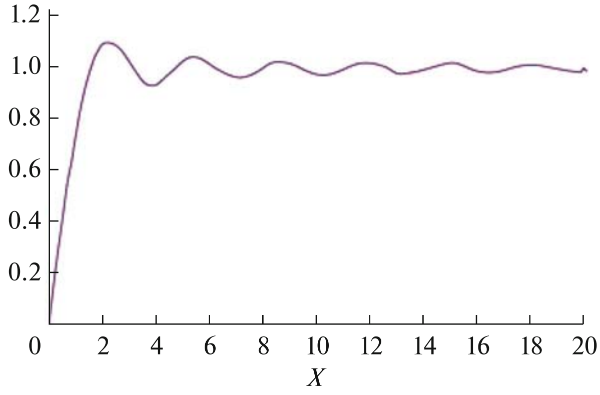

Fig. 1. Graph of the function $\sqrt{1-\frac{\sin 2x}{2x}}$.

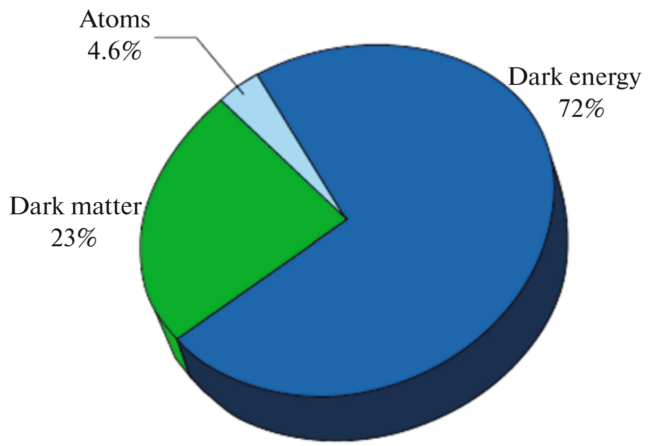

Fig. 2. Composition of the Universe [9].

In deriving $\lambda(r)$ (10), we used the identity:

$$\left(\frac{\sin(ar)}{ar}\right)^2-\frac{\sin(2ar)}{ar}+\cos(2ar)=\frac{d}{dr}\left(\frac{\sin(2ar)}{2a}-\frac{\sin^2(ar)}{a^2r}\right).$$

When a star rotates around the center of galaxy, the centripetal acceleration $\frac{c^2}{2}\frac{d\nu}{dr}$ is balanced by the centrifugal acceleration $\frac{V^2}{r}$. From formulas (9) and (10) it is obtained that the velocity of a star's motion $V(r)$ as a function of a radius r asymptotically reaches a plateau with damped oscillations [3]:

$$V(r)=c\sqrt{\frac{1}{2}r\frac{d\nu}{dr}}=V_{\rm plat}\sqrt{1-\frac{\lambda\!\!\!^{-}}{2r}\sin\left(\frac{2}{\lambda\!\!\!^{-}}r\right)},\quad r\gg r_{\rm h}. \tag{11}$$

The graph of the function $\sqrt{1-\frac{\sin 2x}{2x}}$ is shown in Fig. 1.

The velocity on the plateau

$$V_{\rm plat}=c\sqrt{\frac{\kappa}{2}\lambda\!\!\!^{-}\phi^{m}_{;m}(0)}, \tag{12}$$

depends on the covariant divergence $\varphi^m_{;m}(0)$ of the longitudinal field at the center, indistinguishable from the divergence $\varphi^m_{;m}(r_{\rm h})$ at the surface of the black hole for $\lambda\!\!\!^{-}\gg r_{\rm h}$. For each specific galaxy, the value of $\varphi^m_{;m}(r_{\rm h})$ depends on the interaction of the longitudinal vector field $\varphi^r(r)$ with the black hole located at the center of the galaxy.

ON THE STATIC STATE OF A BLACK HOLE AT THE GALAXY CENTER

Here it is necessary to note the most significant role of the dark sector. In a vacuum (without the stabilizing effect of dark matter), the equilibrium state of a superheavy black hole is impossible [8]. According to the NASA Pie Chart [9] (see Fig. 2), there is only 4.6% ordinary matter in the Universe.

The remaining 95% includes the so-called dark matter (23%) and dark energy (72%).

It is believed that a black hole is a process of unlimited compression (collapse) of matter under the influence of the dominant force of its own gravitational field [10]. Galaxies with black holes at the center have existed for as long as the Universe. With this slow black-hole evolution, the local equilibrium concentration of particles participating in chemical reactions of conversion between different particles depends on temperature and pressure and does not depend on specific channels of the reaction [11]. To establish a link between dark matter and a black hole, it is necessary to show that there exists an equilibrium state to which a gravitational collapse may lead.

In the process of collapse when the pressure increases, at the next step, the elementary particles of the Standard Model may become dominant after neutrons (Fig. 3).

The Bose–Einstein condensate of massive bosons is the energetically most favorable state of matter at low temperatures. These can be the gauge bosons Z and W, the scalar Higgs boson H, as well as bosonic quasiparticles of paired fermions (Cooper effect [13]).

A wave function of the Bose–Einstein condensate is a scalar field [14]. The Lagrangian of a complex scalar field ψ is

$$L=g^{ik}\psi^*_{,i}\psi_{,k}-U\left(|\psi|^2\right),\quad U(0)=0. \tag{13}$$

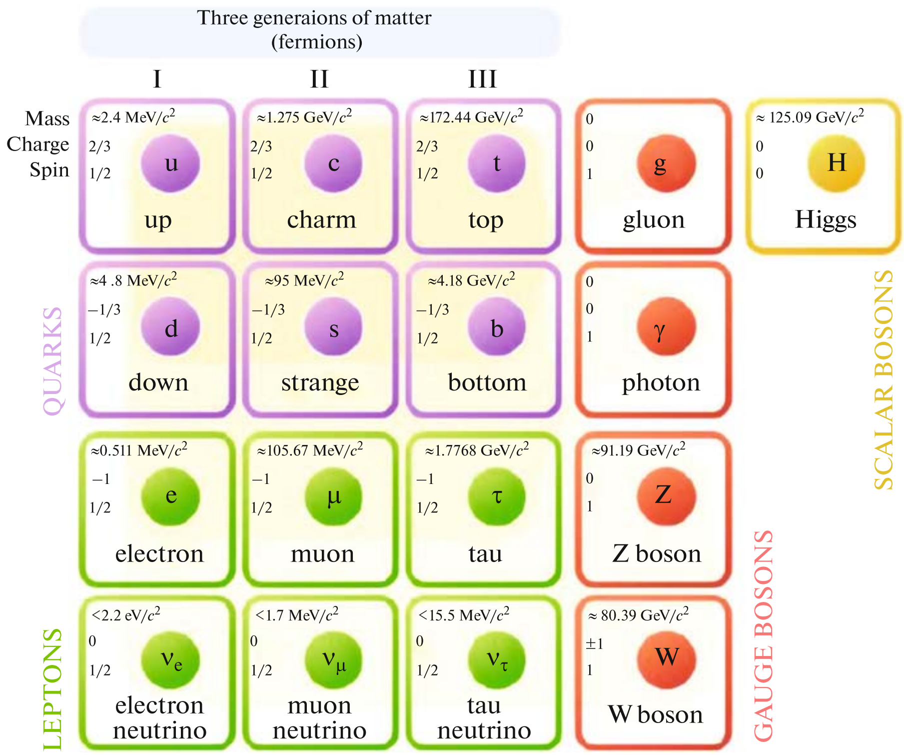

Fig. 3. Standard Model of elementary particles [12].

In the expansion of potential

$$U\left(|\psi^2|\right)=\left(\frac{mc}{\hbar}\right)^2|\psi|^2+\frac{1}{2}\lambda|\psi|^4+\ldots$$

m is the rest mass of a boson. Scalar functions ψ and ψ^* satisfy the Klein–Gordon equation:

$$\frac{1}{\sqrt{\det g_{ik}}}\left(\sqrt{\det g_{ik}}\,g^{lm}\psi_{,l}\right)_{,m}=-\frac{\partial U}{\partial|\psi^2|}\psi. \quad (14)$$

Equation (14) is invariant with respect to a change in sign of $\det g_{ik}$: $\sqrt{-1}$ in the numerator and denominator cancel. Static spherically symmetric scalar field in a state with a certain energy E per particle

$$\psi_E\left(x^i\right)=e^{-iEx^0/\hbar c}\psi(r)$$

formally depends on two coordinates x^0 and r. However, in statics in the equations of Klein–Gordon

$$\begin{aligned} &g^{rr}\psi''+\left(\left(g^{rr}\right)'+\frac{1}{2}\left(\ln\left(\det g_{ik}\right)\right)'g^{rr}\right)\psi' \\ &=\left(\frac{1}{\hbar^2c^2}\left(g^{00}E^2-m^2c^4\right)-\lambda|\psi|^2\right)\psi \end{aligned} \quad (15)$$

and Einstein

$$\left(g^{rr}\right)'+\frac{1+g^{rr}}{r}=\kappa rT_0^0, \quad (16)$$

$$g^{rr}\left(\frac{1}{r}-\left(\ln g^{00}\right)'\right)+\frac{1}{r}=\kappa rT_r^r \quad (17)$$

time x^0 is a cyclic variable. The coordinate x^0 is not included explicitly in the components of energy–momentum tensor T_0^0 and T_r^r, which follow from Lagrangian (13):

$$T_0^0=\frac{1}{\hbar^2c^2}\left(g^{00}E^2+m^2c^4\right)|\psi|^2+\frac{1}{2}\lambda|\psi|^4-g^{rr}|\psi'|^2,$$

$$T_r^r=\frac{1}{\hbar^2c^2}\left(-g^{00}E^2+m^2c^4\right)|\psi|^2+\frac{1}{2}\lambda|\psi|^4+g^{rr}|\psi'|^2.$$

Three equations (one of Klein–Gordon (15) and two of Einstein (16), (17)) for three functions $\psi(r)$, $g^{00}(r)$ and $g^{rr}(r)$ determine the static state of the gravitating Bose–Einstein condensate.

The metric component $g^{rr}(r)$ is the coefficient at the highest derivative of Klein–Gordon equation (15). From the point of view of the existence and uniqueness theorem [15], at gravitational radii $r=r_g$ and $r=r_h>r_g$ (for which in the Schwarzschild metric $g^{rr}(r)=0$) the solution $\psi(r)$ exists, but is not unique. For an arbitrarily large black-hole mass, the presence of an internal gravitational radius r_g ensures the existence of a static solution that is regular at the center [8]. A sphere with the gravitational radius r_h is a boundary of a black hole with the dark matter. The nonuniqueness of solutions with boundary conditions

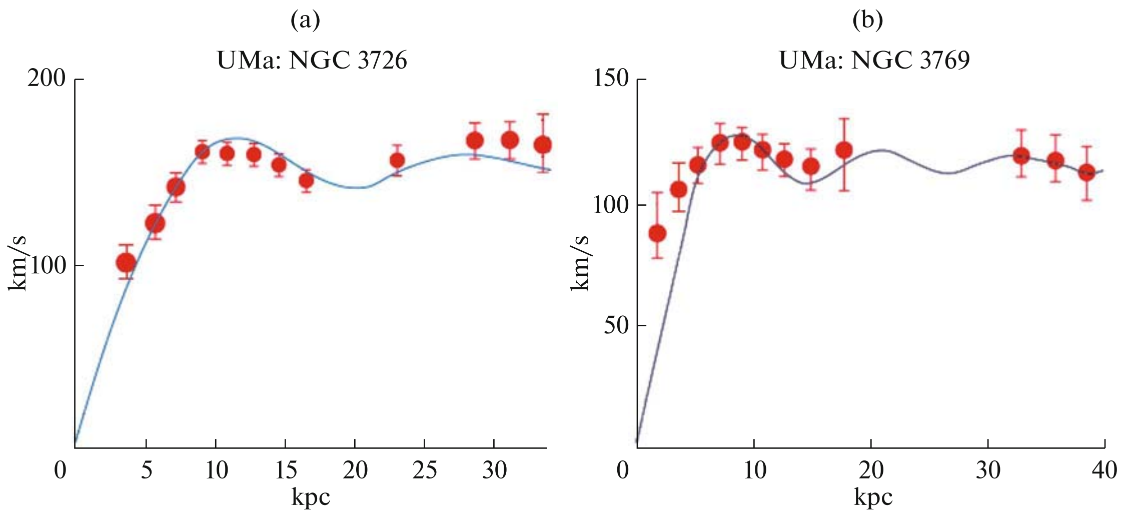

Fig. 4. Rotation curves of (a) NGC 3726 and (b) NGC 3769 spiral galaxies in the Ursa Major cluster [16].

at gravitational radii r_g and r_h confirms the possibility of the existence of a regular static state of a massive black hole in a dark matter halo.

From Einstein's equations (16) and (17), it follows that at the boundary $r = r_h$ the components of energy–momentum tensor $T_0^0(r_h) = 0$ and $T_r^r(r_h) = 1/\kappa r_h^2$. The covariant divergence of a vector field is a scalar that satisfies Klein–Gordon Eq. (6). The scalar wave function of the Bose condensate also satisfies Klein–Gordon equation (15), but only with a different mass of a quantum. One can joke that a divergence of the longitudinal field of dark matter in the region $r > r_h$ is as if the "turned inside out" wave function of the Bose condensate inside the black hole $r < r_h$. A condition of pressure continuity at the interface makes it possible to determine the dependence of the plateau velocity of the galaxy's rotation on the black hole mass [8]:

$$V_{\text{plan}} = c\frac{M_{\text{Pl}}^2}{4\sqrt{\mu m M}}. \qquad (18)$$

Here $M_{\text{Pl}} = \sqrt{\hbar c/k} = 2.177\times 10^{-5}$ g is the Planck mass; M is the mass of a black hole; μ and m are the rest masses of the quanta of longitudinal vector field of dark matter and the bosons of wave function of the black hole condensate; $k = 6.67 \times 10^{-8}$ cm^3/g s^2 is the gravitational constant.

Figure 4 shows the rotation curves of two spiral galaxies NGC 3726 (Fig. 4a) and NGC 3769 (Fig. 4b) in the Ursa Major cluster [16] (UMa is the Ursa Major cluster). The acronym NGC stands for New General Catalog of Nebulae and Star Clusters. The ordinate is the velocity V in km/s, and the abscissa is the distance r from the galactic center in kpc. Points with error bars are the observations. Regarding (18), the solid curves are approximations according to the following formula:

$$V(r) = c\frac{M_{\text{Pl}}^2}{4\sqrt{\mu m M}}\sqrt{1 - \frac{\lambda\!\!\!^{-}}{2r}\sin\left(\frac{2}{\lambda\!\!\!^{-}}r\right)}. \qquad (19)$$

Galaxy NGC 3726 (in Fig. 4a) has a plateau velocity $V \approx 150$ km/s and a de Broglie wavelength $\lambda\!\!\!^{-} \approx 16$ kpc. The rest mass of the quantum of dark matter for this galaxy is $\mu = \hbar/c\lambda\!\!\!^{-} \approx 0.76\times 10^{-60}$ g. The NGC 3769 galaxy in Fig. 4b has the plateau velocity $V \approx 120$ km/s and the wavelength $\lambda\!\!\!^{-} \approx 13$ kpc. The rest energy of massive bosons of the Standard Model of Elementary Particles (see Fig. 3) is around 100 GeV. For quantitative estimations, we will assume that the rest mass of black hole bosons $m \approx 1.78 \times 10^{-22}$ g. It turns out that the masses of black holes at the centers of these galaxies are $M_{3726} \approx 2 \times 10^{34}$ g and $M_{3769} \approx 2.3 \times 10^{34}$ g. The accuracy of estimation of masses of these two black holes is low because it is not clear exactly what kind of bosons make up the Bose–Einstein condensate.

GRAVITATIONAL FIELD OF A BLACK HOLE IN A DARK MATTER HALO

Outside a black hole $r > r_h$, Einstein equation (5)

$$\lambda' + \frac{\lambda}{r} = \kappa r\left[\left(\frac{1}{\lambda\!\!\!^{-}}\varphi^r\right)^2 - \left(\varphi^m_{;m}\right)^2\right]$$

is a linear inhomogeneous ordinary differential equation. Its full solution consists of a sum of the general solution to the homogeneous equation (without the right-hand side) and a particular solution to the inho-

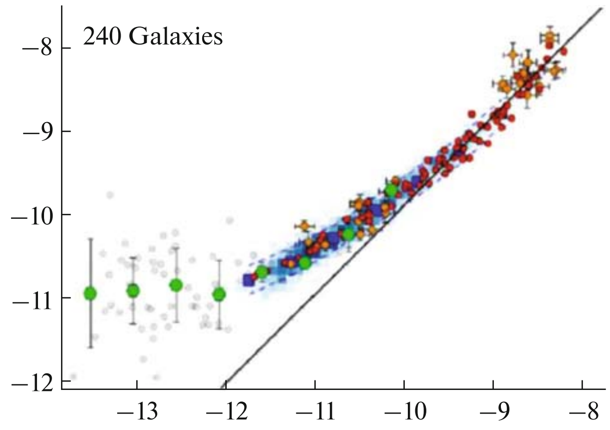

Fig. 5. Deviation of the centripetal acceleration from the Newtonian one observed in 240 different galaxies [17].

mogeneous equation. The particular solution to inhomogeneous equation (10) is:

$$\lambda(r) = 2\frac{V_{\mathrm{plat}}^2}{c^2}\left[\frac{\bar{\lambda}}{2r}\sin\left(\frac{2}{\bar{\lambda}}r\right) - \left(\frac{\bar{\lambda}}{r}\right)^2\sin^2\left(\frac{r}{\bar{\lambda}}\right)\right],$$
$$r > r_{\mathrm{h}}$$

and it determines the contribution of dark matter to gravity. The general solution to the homogeneous equation $\lambda' + \frac{\lambda}{r} = 0$, in our case, is the Schwarzschild solution $\lambda(r) = r_{\mathrm{h}}/r$ [6]. This is the contribution of a black hole to the gravitational field in the region $r > r_{\mathrm{h}}$ that is occupied by the dark matter. Actually, the dependence of the star velocity

$$V(r) = \sqrt{V_{\mathrm{plat}}^2\left(1 - \frac{\bar{\lambda}}{2r}\sin\left(\frac{2}{\bar{\lambda}}r\right)\right) + c^2\frac{r_{\mathrm{h}}}{2r}} \qquad (20)$$

on the mass M of the black hole located at the center of the galaxy manifests itself in two ways. By means of the dark matter alone, the plateau velocity (18) would have decreased with an increase in the black hole mass as M^{-1}. However, only due to the attraction to the black hole (in view of $r_{\mathrm{h}} = \frac{2kM}{c^2}$) the star velocity $V(r)$ would have increased with an increase in mass as $\sim\sqrt{M}$. At distances from the center $r \sim \bar{\lambda}$, the contributions to the gravity jointly from the black hole and the dark matter (the two terms under the root in (20)) turn out to be of the same order with the black hole mass

$$M \sim \tilde{M} \equiv \frac{M_{\mathrm{Pl}}^2}{(16m\mu^2)^{1/3}}.$$

When $M \gg \tilde{M}$ the star velocity $V(r)$ decreases proportionally to $1/\sqrt{r}$, as according to Newton's theory. And vice versa, with a black hole mass $M \ll \tilde{M}$ the curve of rotation of galaxy stars $V(r)$ reaches a plateau. With the rest energy of bosons ~100 GeV of the condensate (with the mass $m \approx 1.78 \times 10^{-22}$ g) and with the mass of quanta of the longitudinal vector field $\mu = \hbar/c\bar{\lambda} \approx 0.76\times10^{-60}$ g, we obtain $\tilde{M} \approx 4 \times 10^{37}$ g.

Figure 5 shows a comparison of the observed (ordinate) centripetal acceleration with the Newtonian one (abscissa) for 240 different galaxies [17]. Without the dark matter, all points would have lain along a straight line at an angle of 45 deg from the axes. For stars of different galaxies moving in a circle of the same radius, the accelerations are proportional to the masses of black holes at the centers of their galaxies. Therefore, the logarithms of black hole masses are plotted along the axes.

The masses of the spiral galaxies NGC 3726 and NGC 3769 of the Ursa Major constellation, $M_{3726} \approx 2 \times 10^{34}$ g and $M_{3769} \approx 2.3 \times 10^{34}$ g, are much less than $\tilde{M} \approx 4 \times 10^{37}$ g. Their place is at the bottom of Fig. 5 on the left. The mass of the black hole at the center of our Milky Way galaxy $M_{\mathrm{MW}} = 8.6 \times 10^{39}$ g is two orders of magnitude greater than $\tilde{M}$. Among the 240 galaxies in Fig. 5, the place of our Milky Way is in the upper right corner.

CONCLUSIONS

Dark matter does not play a significant role in the Milky Way galaxy.

ABBREVIATIONS AND NOTATION

GTR general theory of relativity
NGC New General Catalog of nebulae and star clusters
UMa Ursa Major cluster

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CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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