Capillary propagation of sound and anomalous Kapitsa jump at the boundary between solid and liquid helium

V. I. Marchenko and A. Ya. Parshin
Institute of Physical Problems, USSR Academy of Sciences

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It is shown that the probability of sound propagation across the quantum boundary between liquid and solid helium at $T=0$ is proportional to the square of the frequency. The Kapitsa jump at such a boundary depends, therefore, on the temperature according to a T^{-5} law.

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The boundary between solid and liquid He^4 at $T \lesssim 1$ K has unique properties.[1,2] Only two faces of a helium crystal, identifiable with respect to symmetry, are in the classical state of atomically smooth surface. The quantum delocalization of the surface defects on the remaining faces is so large that a new state,[4] a quantum analog of an atomically rough surface, is produced. The growth and fusion of such boundaries at $T=0$ occur without dissipation—in a coherent manner, and the conditions for a phase equilibrium at the boundary are satisfied at each moment of time in this case. Castaing and Nozières[3] noticed that this leads to an anomalous reflection of sound from the boundary, since the conditions for a phase equilibrium correspond to a specified pressure and the boundary is always adjusted to satisfy this condition by recrystallization. At finite temperatures the thermal excitations retard the motion of the boundary, the conditions for the phase equilibrium break down and the sound passes from one phase to another. Castaing *et al.*[4] recently reported that the propagation of sound decreases with decreasing temperature.

In this paper we show that at $T=0$ the sound passes through quantum boundaries as a result of the capillary effects. The probability of such a passage is proportional to the square of the sound frequency.

Since our only aim is to demonstrate this effect, we shall limit ourselves to the examination of an isotropic crystal. The thermodynamic condition for a Gibbs phase equilibrium in this case has the form

$$F_0 + P + \alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\mu}{v_0} \tag{1}$$

where F_0 is the free energy of a unit volume of the undeformed crystal, v_0 is its atomic volume, μ is the chemical potential of the liquid, P is the pressure of the liquid, α is the density of the surface energy, and R_1 and R_2 are the main radii of curvature. The pressure P in an acoustic wave differs from the equilibrium pressure P_0 by an amount δP. Since $\mu(P) = \mu(P_0 + \delta P) \approx \mu(P_0) + v\delta P$, where v is the atomic volume of the liquid, we obtain from Eq. (1)

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$$\left(\frac{v}{v_0}-1\right)\delta P=a\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \tag{2}$$

The conditions for a mechanical equilibrium in the isotropic case reduce to the following equations (for details see Ref. 5):

$$\sigma_{nn}+P+\beta\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=0,\quad \sigma_{\mu n}=0 \tag{3}$$

where σ_{ik} is the stress tensor in the crystal, n is the index of the normal to the surface, $\mu=1,2$ are the indices of the Cartesian coordinate system in the boundary plane, and β is the surface tension coefficient.

The problem can be fully defined by adding one more condition to the system of boundary conditions (2) and (3), a corollary of the conservation of particles

$$\dot{u}_n-V_n=\dot{\zeta}\left(\frac{v}{v_0}-1\right), \tag{4}$$

which, in the absence of recrystallization ($\dot{\zeta}=0$), reduces to the equality of the normal velocity components of the liquid V_n and solid $\dot{u}_n$. The value of ζ represents the boundary shift due to pressure or crystallization.

As a result of emission of an acoustic wave, for example, from a liquid, ripples are formed on the boundary due to recrystallization $\zeta\sim\exp(ikx-i\omega t)$ with a wave vector k_x equal to the tangential projection of the wave vector of sound $\mathbf{k}$, where $R_2^{-1}=0$, but $R_1^{-1}\approx\partial^2\zeta/\partial x^2$ and, according to the conditions (3), a voltage is produced in the crystal, i.e., the sound penetrates the crystal. In determining the passage of sound from the liquid to the crystal, the velocity $\dot{u}_n\ll V_n$ should be disregarded in Eq. (4) and the $V_n\ll\dot{u}_n$ should also be disregarded in determining the passage of sound from the crystal to the liquid. Without dwelling on the standard calculations (see Ref. 6), we give, for example, the expression for the ratio of the amplitude A_t of the transmitted transverse sound to the amplitude A of sound emitted from the liquid at an angle θ to the normal:

$$\frac{A_t}{A}=2i\,\frac{c_t^2(1+\sigma)(1-2\sigma)\left\{a+\left(\frac{v}{v_0}-1\right)\beta\right\}\cos\theta\sin^2\theta}{c^3E\left(\frac{v}{v_0}-1\right)^2\left\{(1-2\sigma)\sin^2\theta_t\cos\theta_t+(\cos^2\theta_l-\sigma)\dfrac{\cos 2\theta_t}{\sin 2\theta_l}\right.}\,\omega, \tag{5}$$

where c is the velocity of sound in a liquid and the remaining notations (c_t, σ, E, θ_t, and θ_l) are standard notation in the theory of elasticity (see Refs. 6 and 7).

Thus, the probability of the passage of sound, equal to the square of the modulus of the amplitude ratio (5), is proportional to the square of the frequency ω^2.

It is clear that this greatly complicates the heat transfer at the boundary of liquid-solid helium at temperatures $\lesssim 1$ K and the Kapitsa jump here should have an anomalous temperature dependence T^{-5}.

We note that there are ordinary Rayleigh waves on the quantum surfaces of a helium crystal whose spectrum is determined only by the properties of the crystal, like the boundary with the vacuum. An allowance for the capillary effects leads to their velocity dispersion but not damping, since the sound velocity in liquid helium is larger than the velocity of transverse waves in solid helium. The velocity of surface acoustic waves on both classical faces also depends substantially on the properties of the liquid (see Ref. 6). The heat transfer across such boundaries is achieved in the usual way.

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