Theory of localization in large-dimensionality spaces

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The divergence of the localization length does not lead to a divergence of the dielectric constant in the case of a Bethe lattice. This conclusion resolves the contradiction between two exact results which have been published. Comparison of these results indicates that there is a difference in the correlation lengths above and below the Anderson transition. This circumstance imposes severe constraints on the possible nature of the theory.

Systematic formulations of localization theory are available only for spaces of dimensionalities $d = 2 + \epsilon$ (Ref. 1) and $d = \infty$ (Refs. 2 and 3). Study of the case $d = \infty$ has become particularly urgent because of the difficulties which the $2 + \epsilon$ theory has recently encountered.[4,5]

A realization of the limit $d = \infty$ is a Bethe lattice, which is a branched tree (a Cayley tree). A graphic way to interpret this lattice is as a cluster on a d-dimensional lattice, where d must tend toward infinity along with the thermodynamic limit. The validity of this interpretation is supported by our entire experience in the theory of phase transitions; the extensive results for a Bethe lattice agree with the results for d-dimensional lattices if the former is assigned an effective dimensionality $d = \infty$.

Two groups of results have now been published for a Bethe lattice: 1) The results on the existence of a minimum metallic conductivity $\sigma_{\min}$ and a maximum dielectric constant $\epsilon_{\max}$ obtained by Efetov *et al.*[2] 2) The result $\nu = 1/2$ for the critical index of the localization radius obtained by Kunz and Souillard.[3] In each case, the authors claim an exact solution of the corresponding model[1)]; furthermore, the result $\nu = 1/2$ and the existence of $\sigma_{\min}$ are supported by numerical calculations.[6,7] According to the general understanding, however, the result $\nu = 1/2$ contradicts the existence of a $\epsilon_{\max}$, since a divergence of the localization radius would imply a divergence of the dielectric constant. In the present letter we show how this contradiction is resolved.

We begin with the localization criterion derived by Thouless.[8] We partition the system into blocks of dimension L, and we introduce the parameter g_L as the ratio of the scale value of the overlap integral between blocks to the scale value of the spread of levels in adjacent blocks. In order of magnitude, g_L is the total (not the specific) conductivity of a block of dimension L in units of $e^2/\hbar$. The dependence of g_L on L for the various values g_0 which g_L assumes at microscopic scale, l, must be as follows (Fig. 1): $g_L \sim 1$ at the point of the Anderson transition, $g_0 = g_0$; $g_L \to \infty$ in the limit $L \to \infty$ in the region of delocalized states ($g_0 > g_c$) and $g_L \to 0$ in the localized phase ($g_0 < g_c$). This criterion is based on a clear physical idea: At large values of g_L, the wave functions of the different blocks are mixed with roughly equal weights, while at small values of g_L there is essentially no mixing.

We introduce the correlation lengths ξ and ξ' for $g_0 < g_c$ and $g_0 > g_c$, respectively, as scale distances over which the deviation of g_L from g_c reaches a value $\sim g_c$, and we

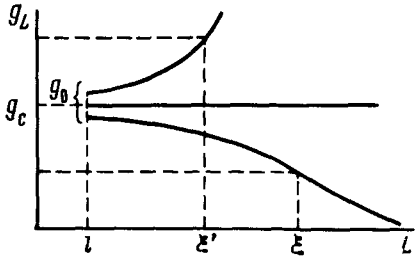

FIG. 1.

introduce critical indices for ξ, ξ' and the conductivity σ:

$$\xi \sim (g_c - g_0)^{-\nu}, \quad \xi' \propto (g_0 - g_c)^{-\nu'}, \quad \sigma \propto (g_0 - g_c)^s . \tag{1}$$

In general, there is no reason to assume that the indices ν and ν' are equal.

For the indices s and ν' we have the relation

$$s = \nu'(d-2). \tag{2}$$

For $L \lesssim \xi'$ we have $g_L \sim g_c$ by virtue of the definition of ξ'; for $L \gg \xi'$ we have $g_L \gg 1$, and the disordered system is a good metal with a macroscopic specific conductivity σ, so that we have $g_L \sim \sigma L^{d-2}$. Combining these two relations for $L \sim \xi'$, we find $\sigma \sim g_c \xi'^{(2-d)}$ and thus (2).

Using Efetov's first result,[2] $s = 0$, we find $\nu' = 0$; using the result of Kunz and Souillard,[3] we find $\nu = 1/2$. We can show that this result does not contradict Efetov's second result: the existence of an $\epsilon_{\max}$.

The conclusion that there is a divergence of the dielectric constant at the transition point is based on the following considerations. A disordered system in a localized phase may be represented as a system of metallic granules of size ξ which are separated by a dielectric layer. For wave vectors $g \gtrsim 1/\xi$, the dielectric constant is then of the ordinary metallic nature: $\epsilon(q) \sim 1/q^2$. We thus find $\epsilon(1/\xi) \sim \xi^2$, and letting ξ go to infinity, we find $\epsilon(0) = \infty$. Let us analyze this argument more carefully.

At small values of q and ω, we can use the following expansion for the longitudinal dielectric constant of a metal:

$$\epsilon(q, \omega) = \frac{4\pi\sigma}{-i\omega + Dq^2} + \epsilon_0 + \dots , \tag{3}$$

where the diffusion coefficient D is related to σ by the Einstein relation $\sigma = e^2 D\nu(\mu)$, and $\nu(\mu)$ is the state density at the Fermi level. Assuming $q \sim 1/\xi$, we find the dielectric constant of the disordered system to be

$$\epsilon(1/\xi, \omega) = \frac{4\pi\sigma_\xi}{-i\omega + D_\xi \xi^{-2}} + \epsilon_0 , \tag{4}$$

where σ_ξ and D_ξ are the conductivity and diffusion coefficient of a granule of size ξ. Since at $L \lesssim <\xi$ we have $g_L \sim g_c$ by virtue of the definition of ξ, we can write

$$\sigma_\xi \sim g_c / \xi^{d-2}$$

At large values of ξ, we find $\sigma_\xi \to 0$ as $d \to \infty$.

An important point is that the limit $\sigma_\xi \to 0$, $\omega \to 0$ in (4) depends on the order in which the limits are taken. A lower bound is imposed on the frequency ω by the reciprocal of the measurement time, and the order in which the limits are taken is determined from the causality principle: First the object of the measurement is selected, and then the measurement is carried out. If the object of the measurement is a d-dimensional lattice, the limit $\omega \to 0$ can be taken first, and the static dielectric constant diverges as $\xi \to \infty$. If the object of the measurement is a Bethe lattice, the limit $d \to \infty$ is taken before the beginning of the measurements, so that the limit $\sigma_\xi \to 0$ is taken before the limit $\omega \to 0$. The dielectric constant is therefore equal of ϵ_0 and remains finite in the limit $\xi \to \infty$. The existence of a maximum dielectric constant is thus a specific feature of a Bethe lattice and is not a property of a space of finite dimensionality.

The relation $\nu = \nu'$ is known to follow from the hypothesis of one-parameter scaling[9] by virtue of the assumption of the analyticity of the Gell-Mann–Low function, which is the only reasonable assumption within the context of one-parameter scaling. The results of Refs. 2 and 3 thus unambiguously rule out one-parameter scaling in spaces of high dimensionality.[2)]

Another consequence of the inequality $\nu \neq \nu'$ is that it is not possible to construct a mean-field theory in the form of the Landau theory for an Anderson transition. Correspondingly, the ϵ expansion near the upper critical dimensionality (if it exists) cannot have the standard form of the theory of critical phenomena. This conclusion rules out, in particular, approaches based on a reduction of the localization problem to a percolation.

In summary, for high-dimensionality spaces the values offered by Mott[10] more than 20 years ago for the indices, $s = 0, \nu = 1/2$, are correct. Whether they are correct all the way to $d = 2$ or to some upper critical dimensionality remains an open question.

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1)The models used in Refs. 2 and 3 differ in secondary details which are unimportant to the critical behavior.
2)The result $s = \nu' = 0$ might itself arise asymptotically in the limit $d \to \infty$, without disturbing the existence of one-parameter scaling at finite d.

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