Theory of spin ordering in metals

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The exchange symmetry of the helical and spin–split states of metals is clarified. Each structure is a spin nematic.

When the Pomeranchuk conditions[1] for an electron Fermi liquid are violated in a metal, some state of lowered symmetry should prevail. The simplest example is a ferromagnet (Fig. 1a). Akhiezer and Chudnovskiĭ[2] pointed out that a special spin order—helical—might occur (Fig. 1b). Recently Hirsch[3] discussed yet another structure, analogous to a helical structure: a spin–split state (Fig. 1c).

In a ferromagnet, the spin dynamics is described by the Landau–Lifshitz equation. In other cases, the spin dynamics and the magnetic properties are generally still open questions. Not all of the states which have been described could literally be eigenstates of any reasonable Hamiltonian, but Fig. 1 reflects the symmetry of the spin order. In these states, relativistic effects are assumed small, so the properties of these states should be determined completely by the exchange symmetry.[4–6]

In structure *b*, the isotropy of the spin space is completely disrupted, while in structure *c* an axial symmetry remains. Symmetry under time reversal is retained in both states. Spin nematics have these properties.[5] To verify that the states we are dicussing are indeed spin nematics, let us examine the nature of the spin correlations in them.

We begin with the simpler case, in Fig. 1c. We find the correlation function $s_{xy}(\vec{t}) = \langle s_x(0)s_y(\vec{r})\rangle$ (the z axis in spin space runs along the "axial" axis). Using the relationships

$$s^+ = a_\uparrow^+ a_\downarrow; \quad s^- = a_\downarrow^+ a_\uparrow; \quad s_z = (a_\uparrow^+ a_\uparrow - a_\downarrow^+ a_\downarrow)/2 \tag{1}$$

between the spin operators $s^\pm = s_x \pm is_y$, s_z and the operators which create and annihilate electrons with certain z projections of the spin, we find

$$s_{xy}(\vec{r}) = \frac{1}{4i} < (a_\uparrow^+ a_\downarrow + a_\downarrow^+ a_\uparrow)_0 (a_\uparrow^+ a_\downarrow - a_\downarrow^+ a_\uparrow)_{\vec{r}} >$$
$$= \frac{1}{4i} \sum < (a_{\uparrow\vec{k}1}^+ a_{\downarrow\vec{k}2} + a_{\downarrow\vec{k}1}^+ a_{\uparrow\vec{k}2})_0 (a_{\uparrow\vec{k}3}^+ a_{\downarrow\vec{k}4} - a_{\downarrow\vec{k}3}^+ a_{\uparrow\vec{k}4})_{\vec{r}} > e^{i(\vec{k}1-\vec{k}2)\vec{r}}$$
$$= \frac{1}{2} \sum n_{\uparrow\vec{k}} n_{\downarrow\vec{q}} \sin(\vec{k} - \vec{q})\vec{r}. \tag{2}$$

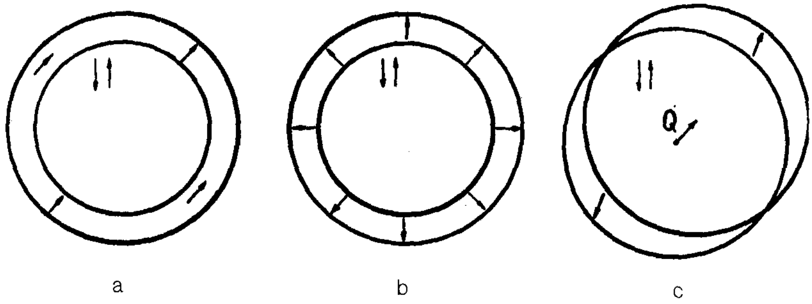

FIG. 1.

We now change variables: $\vec{k}\to\vec{k}+\vec{Q}/2$ and $\vec{q}\to\vec{q}-\vec{Q}/2$, where $\vec{Q}$ is the relative shift of the Fermi spheres of electrons with opposite spins (Fig. 1c). We then find

$$s_{xy}(\vec{r}) = \frac{1}{2}\sum n_{\uparrow\vec{k}+\vec{Q}/2}n_{\downarrow\vec{q}-\vec{Q}/2}\sin(\vec{k}-\vec{q}-\vec{Q})\vec{r}$$

$$= \frac{1}{2}\sin\vec{Q}\vec{r}\sum_{k} n_{\uparrow\vec{k}+\vec{Q}/2}\cos\vec{k}\vec{r}\sum_{q} n_{\downarrow\vec{q}-\vec{Q}/2}\cos\vec{q}\vec{r}$$

$$= \pi\sin\vec{Q}\vec{r}\{\int_0^{k_F}\int_0^{\pi}\cos(qr\cos\theta)\frac{k^2dk}{(2\pi)^3}\sin\theta d\theta\}^2. \qquad (3)$$

Here we have made use of the relationship $n_{\uparrow\vec{k}+\vec{Q}/2}=n_{\downarrow\vec{q}-\vec{Q}/2}=n_{\vec{k}}$ in state c, where $n_{\vec{k}}$ is a Fermi distribution with a Fermi momentum k_F. At large distances $r\gg k_F^{-1}$ we find

$$s_{xy}(\vec{r}) = \frac{1}{32\pi^4}\frac{k_F^2}{r^4}\cos^2 k_F r\sin\vec{Q}\vec{r}. \qquad (4)$$

It is a simple matter to prove

$$s_{xy}(\vec{r}) = <s_x(0)s_y(\vec{r})> = -<s_x(\vec{r})s_y(0)> = -s_{yx}(\vec{r}). \qquad (5)$$

The antisymmetric part of the spin correlation function has the following form for an arbitrary choice of coordinates:

$$s_{\alpha\beta}(\vec{r}) = e_{\alpha\beta\gamma}P_\gamma\sin\vec{Q}\vec{r}\Phi(r), \qquad (6)$$

where the function $\Phi(r)$ is not changed by the symmetry elements. The pseudovector P_γ is the order parameter of one of the spin nematics studied by Andreev and Grishchuk.[5] In this state the spin dynamics is described[5] by the same equations as for the dynamics of collinear antiferromagnets (§4 in Ref. 4).

State c was proposed by Hirsch[3] for describing the order in chromium. In a spin nematic, however, symmetry under time reversal means that there cannot be a hyperfine field. A hyperfine field has been observed in chromium (see the review by Fawcett[8]). We might add that Hirsch proposed the existence of a spontaneous spin current. There are no objections to this suggestion from the symmetry standpoint, but by evaluating the expression given in Ref. 3 for the spin current exactly one can easily verify that the spin current is zero at equilibrium. Here, as in an analysis of the electric current permitted by the symmetry in structure *b* in an external magnetic field, the expression for the spin current reduces to a total derivative [cf. the derivation of Eq. (19) in Ref. 9].

In case *b*, it is better to go over to the operators $a_{\vec{k}+,-}$, for electrons with a definite helicity:

$$a_{\vec{k}\uparrow} = e^{i\varphi/2}\cos\frac{\theta}{2}a_{\vec{k}+} - e^{i\varphi/2}\sin\frac{\theta}{2}a_{\vec{k}-}\,,$$
$$a_{\vec{k}\downarrow} = e^{-i\varphi/2}\sin\frac{\theta}{2}a_{\vec{k}+} + e^{-i\varphi/2}\cos\frac{\theta}{2}a_{\vec{k}-} \tag{7}$$

[cf. Eqs. (23) and (14) in Ref. 7]. In this case the correlation function does not have a part which is antisymmetric in terms of spin indices. For the difference between average values $\langle s_z(0)s_z(\vec{r})\rangle - \langle s_x(0)s_x(\vec{r})\rangle$, where the z axis runs long $\vec{r}$, we find

$$\Big|\sum_{\vec{k}} e^{i\vec{k}\vec{r}}\cos\theta(n_{\vec{k}+} - n_{\vec{k}-})\Big|^2 = \frac{1}{4\pi^4}\frac{k_F^2}{r^4}\cos^2 k_F r\sin^2 Qr. \tag{8}$$

The quantity $Q \ll k_F$ determines the shift of the Fermi momenta for particles with a definite helicity with respect to the average value k_F. The spin dynamics in this state is described by equations characteristic of a noncollinear antiferromagnetic material.[4]

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