Critical Exponents from Field Theory: New Evaluation.

A. A. Pogorelov, I. M. Suslov
P.L.Kapitza Institute for Physical Problems,
119337 Moscow, Russia
E-mail: suslov@kapitza.ras.ru

Abstract

We present new evaluation of the critical exponents of $O(n)$–symmetric ϕ^4 theory from the field theoretical renormalization group, based on the new algorithm for summing divergent series. The central values practically coincide with those by Le Guillou and Zinn-Justin (1980) but their uncertainty is essentially smaller.

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1. Introduction

The present paper has an aim to give new evaluation of the critical exponents of $O(n)$–symmetric ϕ^4 theory from the field theoretical renormalization group (RG) [1], based on the new algorithm for summing divergent series [2].

According to the formalism of the field-theoretical RG, one should calculate three functions $\beta(g)$, $\eta(g)$, $\eta_2(g)$ entering the Callan-Symanzik equation, find a non-trivial root g^* of equation $\beta(g) = 0$ (determining the fixed point of the RG equations), and then the critical exponents η and ν, as well as the exponent ω of correction to scaling, are given by expressions

$$\eta = \eta(g^*)\,, \qquad \nu^{-1} = 2 - \eta(g^*) + \eta_2(g^*)\,, \qquad \omega = \beta'(g^*)\,. \tag{1}$$

The RG functions are given by factorially divergent series in powers of the coupling constant g and to calculate them one need a method for summing divergent series. The examples are Pade-Borel [3] or conformal Borel [4] techniques.

Our initial information is the same as in the paper [5], i.e. the first 7 expansion coefficients of the RG functions $\beta(g)$, $\eta(g)$, $\eta_2(g)$ [3, 5] and their large order behavior [6] established in the framework of the Lipatov method [7]. The main difference from the preceding papers consists in the fact that explicit interpolation of the coefficient function is made from the very beginning: the low order expansion coefficients are smoothly interpolated with their large order asymptotics and unknown intermediate coefficients are found in a certain approximation. Considering these coefficients as exact, one can sum the divergent series with (in principle) arbitrary precision. The only uncertainty of the algorithm is related with ambiguity of interpolation, which has a clear physical sense and originates from incompleteness of initial information. As a consequence, the relation of the summation results with the assumed behavior of the coefficient functions can be constructively analyzed and estimation of uncertainty becomes completely transparent. For technical reasons, such procedure was impossible in the conventional algorithms due to catastrophic increase of errors in the course of the series resummation, which made interpolation to be useless. A crucial point is stability of our algorithm with respect to smooth errors, involving ambiguity of interpolation.

2. Summation procedure

Our summation procedure [2] is based on the fact that the divergent series

$$W(g) = \sum_{N=N_0}^{\infty} W_N(-g)^N \tag{2}$$

whose coefficients have asymptotic behavior $W_N^{as} = c\, a^N \Gamma(N+b)$, after the Borel transformation

$$W(g) = \int_0^\infty dx e^{-x} x^{b_0-1} B(gx)\,, \qquad B(z) = \sum_{N=N_0}^{\infty} B_N(-z)^N\,, \qquad B_N = \frac{W_N}{\Gamma(N+b_0)} \tag{3}$$

(where b_0 is an arbitrary parameter) and conformal mapping [1] $z = u/(1-u)a$ reduces to a convergent power series in u with coefficients

$$U_0 = B_0\,, \qquad U_N = \sum_{K=1}^{N} \frac{B_K}{a^K}(-1)^K C_{N-1}^{K-1} \qquad (N \geq 1)\,, \tag{4}$$

whose asymptotics at $N \to \infty$

$$U_N = U_\infty N^{\alpha-1}\,, \qquad U_\infty = \frac{W_\infty}{a^\alpha \Gamma(\alpha)\Gamma(b_0+\alpha)} \tag{5}$$

is related with the strong coupling asymptotic behavior of the function $W(g)$,

$$W(g) = W_\infty g^\alpha \qquad (g \to \infty)\,. \tag{6}$$

The coefficients U_N for $N \lesssim 40$ are calculated straightforwardly by Eq.(4) and then are continued according to power law (5) in order to avoid the catastrophic increase in errors [2]. Thus, all the coefficients of the convergent series are known and this series can be summed with (in principle) arbitrary accuracy. This completely removes the problem of the dependence of the results on variation in the summation procedure, which is the main

[1] This conformal mapping is different from that used in [4, 5]. Its advantage consists in the more slow growth of random errors in the coefficients U_N of resummed series (4) and "super-stability" of the algorithm with respect to smooth errors [2].

disadvantage of the commonly accepted methods. The interpolation is performed for the reduced coefficient function

$$F_N = \frac{W_N}{W_N^{as}} = 1 + \frac{A_1}{N-\tilde{N}} + \frac{A_2}{(N-\tilde{N})^2} + \ldots + \frac{A_K}{(N-\tilde{N})^K} + \ldots \qquad (7)$$

by truncating the series and choosing the coefficients A_K from the correspondence with the known values of the coefficients W_{L_0}, $W_{L_0+1}, \ldots$, W_L. The Lipatov asymptotics W_N^{as} is taken in the optimal form $W_N^{as} = ca^N N^{b-1/2}\Gamma(N+1/2)$ [2] [2], and the parameter $\tilde{N}$ is used to analyze uncertainty in the results. The L_0 value sometimes does not coincide with N_0 appearing in Eq.2. Indeed, the coefficient function W_N continued to the complex plane has a singularity at the point $N = \alpha$, where α is the exponent of the strong-coupling asymptotics (6) [2]. If the exponent α is larger than N_0, the interpolation with the use of all the coefficients is inapplicable: it is necessary to set

$$W(g) = W_{N_0} g^{N_0} + \ldots + W_{N_1} g^{N_1} + \tilde{W}(g)\,, \qquad N_1 = [\alpha]\,, \qquad (8)$$

produce summation of the series for $\tilde{W}(g)$, and add the contribution from the separated terms; thus, the value $[\alpha]+1$ ($[\ldots]$ is the integer part of a number) is taken for L_0. Analysis of the two-dimensional case [8] shows that α is larger than N_0 for almost all the functions.

One can see that realization of this program includes (at the intermediate stage) determination of the strong coupling asymptotics of the function $W(g)$. A large accuracy of this asymptotics is not necessary for summation in the region $g \sim 1$ and its rough estimation is sufficient.

Following the tradition, we sum the series not only for the functions $\beta(g)$, $\eta(g)$, $\eta_2(g)$, but also for the functions $\nu^{-1}(g) = 2 + \eta_2(g) - \eta(g)$ and $\gamma^{-1}(g) = 1 - \eta_2(g)/(2-\eta(g))$ in order to verify self-consistency of the results. A set of possible interpolations was restricted by two natural requirements [8]: (a) the interpolation curve comes smoothly through the known points and does not have essential kinks for non-integer N; (b) large N asymptotic behavior is reached sufficiently quickly, and nonmonotonity at large N is on the same scale, as a relative difference of the last known coefficient from the Lipatov asymptotics.

3. The polymer case ($n = 0$)

Initial information is given by expansions [3, 5]

$$\beta(g) = -g + g^2 - 0.4398148149 g^3 + 0.3899226895 g^4 - 0.4473160967 g^5 + 0.63385550 g^6$$

$$-1.034928 g^7 + \ldots + ca^N \Gamma(N+b) g^N + \ldots\,,$$

$$\eta(g) = (1/108) g^2 + 0.0007713750 g^3 + 0.0015898706 g^4 - 0.0006606149 g^5 \qquad (9)$$

[2] We tried another parametrizations of the form $W_N^{as} = ca^N N^{\tilde{b}}\Gamma(N+b-\tilde{b})$ but the results were practically the same, if the same principle was used for restriction of the set of interpolations.

$$+0.0014103421 g^6 - 0.001901867 g^7 + \ldots + c' a^N \Gamma(N+b') g^N + \ldots\,,$$
$$\eta_2(g) = -(1/4)g + (1/16)g^2 - 0.0357672729 g^3 + 0.0343748465 g^4 - 0.0408958349 g^5$$
$$+0.0597050472 g^6 - 0.09928487 g^7 + \ldots + c'' a^N \Gamma(N+b) g^N + \ldots\,,$$

with the parameters [6]

$$a = 0.16624600\,, \quad b = b'+1 = 4\,, \qquad c = 0.085489\,, \quad c' = 0.0028836\,, \quad c'' = 0.010107\,. \tag{10}$$

Below we discuss some technical details of the summation procedure.

Function $\beta(g)$. All the interpolations with $L_0 = 1$ are unsatisfactory: the interpolation curves rapidly achieving their asymptotic behavior exhibit a sharp kink in the interval $1 < N < 2$, indicating a singularity in this interval. Estimation of the strong-coupling asymptotics yields $\alpha \approx 1$, confirming the singularity at $N \approx 1$ and indicating that the choice $L_0 = 2$ should be made. In this case, the interpolation curves with $\tilde{N} < -0.9$ exhibit significant non-monotonicity at large N, and the curves with $\tilde{N} > 1.1$ have a kink in the interval $2 < N < 3$ (see Fig.1). Thus, the "natural" interpolations correspond to the interval $-0.9 < \tilde{N} < 1.1$. The summation results are shown in the inset in Fig.1, which indicate that

$$g^* = 1.420 \div 1.426\,, \qquad \omega = 0.784 \div 0.795\,. \tag{11}$$

The g^* value is in agreement with the results of early works ($g^* = 1.421 \pm 0.004$ [3], $g^* = 1.421 \pm 0.008$ [4]) and in a certain conflict with the more recent evaluation $g^* = 1.413 \pm 0.006$ [5].

Function $\eta(g)$. According to Eq. (3), the expansion for $\eta(g)$ begins with g^2. We fail to obtain satisfactory interpolations with $L_0 = 2$: the curves rapidly approaching large N asymptotic behavior exhibit a kink in the interval $2 < N < 3$, indicating that the exponent α lies in the same interval. Indeed, the estimate of strong-coupling behavior gives $\alpha \approx 2$ and suggests the choice $L_0 = 3$. In this case, the satisfactory interpolation curves (see Fig.2,a) exist only for $1.5 < \tilde{N} < 2.2$. They could be considered unsatisfactory due to a kink for $3 < N < 4$; however, the curves of such a shape provide the exact η value in the two-dimensional case [8]. In our opinion, such interpolations are allowable because the amplitude of oscillations of the coefficient function is on the order of the amplitude of oscillations of the known coefficients. The summation results are shown in the inset in Fig.2,a and give $\eta = 0.0269 \div 0.0275$.

Functions $\eta_2(g)$, $\nu^{-1}(g)$ $\gamma^{-1}(g)$. Rough estimates of the strong coupling behavior for $\eta_2(g)$ and $\nu^{-1}(g)$ gives $\alpha \approx 2$ but in general the results are not self-consistent and violate the relations between RG functions. Analysis of the 2D case [8] shows that it is related with specific features of $\eta(g)$: due to small expansion coefficients, this function is small for $g \lesssim 10$, but grows rapidly for large g. As a result, asymptotic behavior of $\nu^{-1}(g)$ and $\eta_2(g)$ contains a mixture of g and g^2 terms, which is difficult to analyze numerically. Therefore, summation of the series for $\eta_2(g)$ and $\nu^{-1}(g)$ is performed at $L_0 = 3$ [3] in order to take into account a

[3] Summation at $L_0 = 2$ gives practically the same results but with lesser uncertainty.

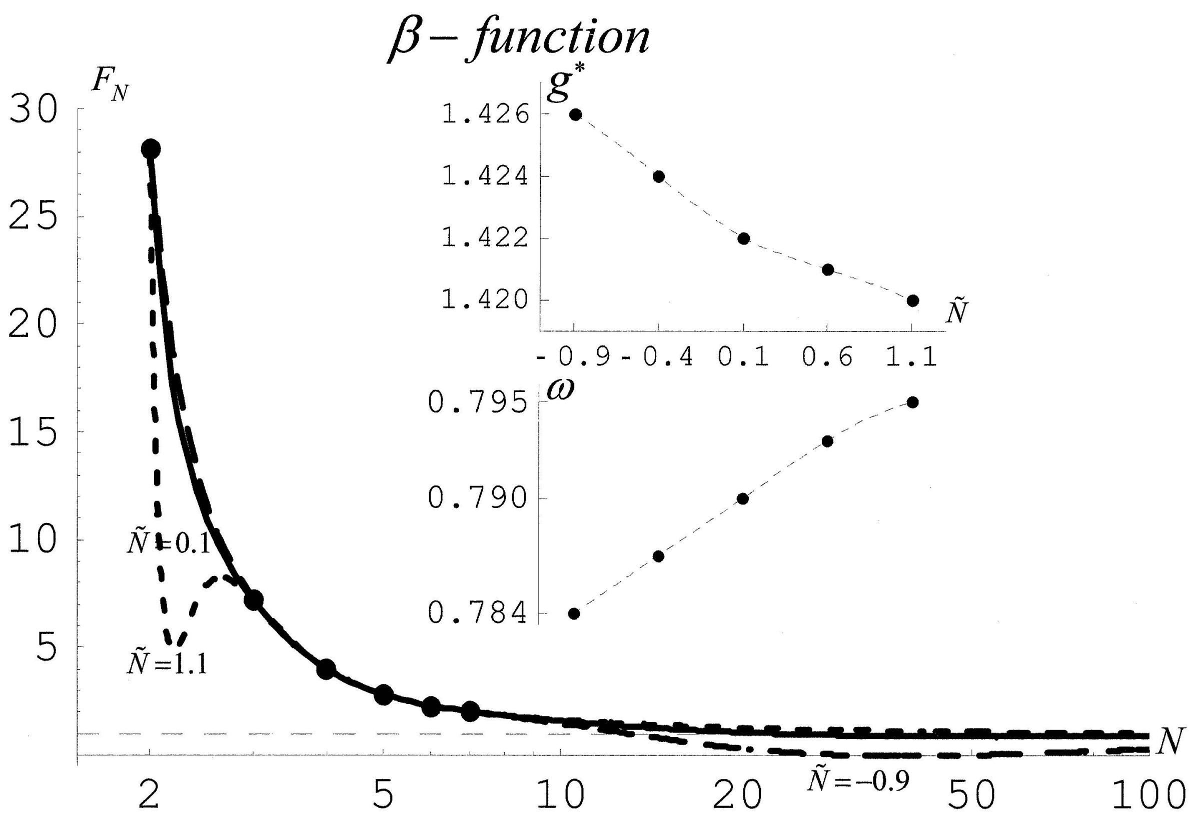

Figure 1: Interpolation curves for the expansion coefficients of $\beta(g)$ and summation results for g^* and ω.

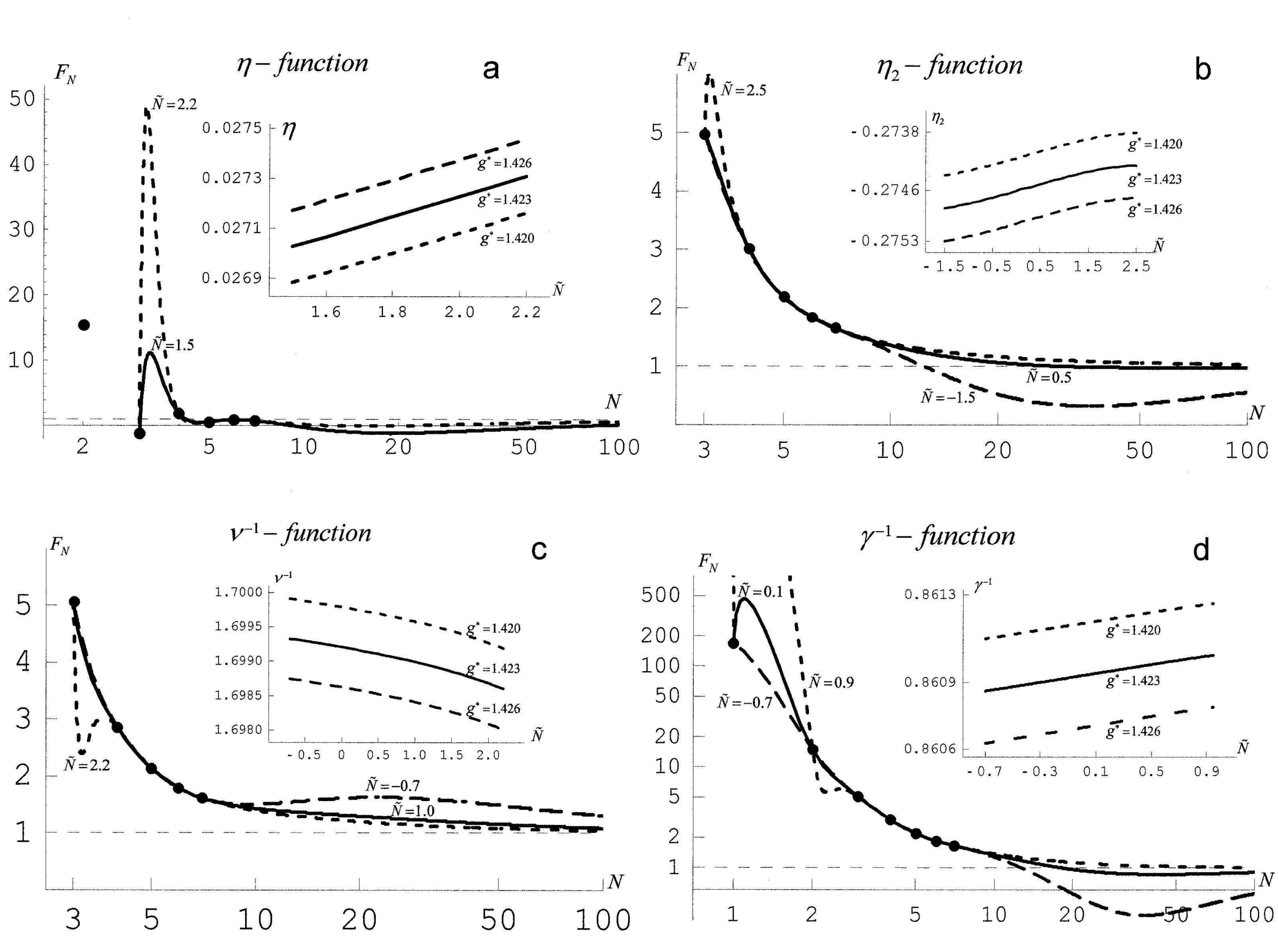

Figure 2: Interpolation curves for the expansion coefficients of functions $\eta(g)$ (a), $\eta_2(g)$ (b), $\nu^{-1}(g)$ (c) and $\gamma^{-1}(g)$ (d). The insets show the summation results at $g = g^*$.

possible singularity at $N \approx 2$, while the series for $\gamma^{-1}(g)$ is summed at $L_0 = 1$, but without the restriction of kinks for non-integer N (due to relation $\gamma^{-1}(g) = 1 + \eta_2(g)/(2 - \eta(g))$ its coefficient function is expected to be regular for $N \geq 1$, but containing a smeared singularity at $N \approx 1$). Figures 2,b–d show the allowable interpolations and summation results. The latter are presented in Table 1 and compared with results by other authors

T a b l e 1.

Critical exponents for the polymer case ($n = 0$) from the field theory

	BNM [3]	LG–ZJ [4]	G–ZJ [5]	Kl [10]	J–Kl [11]	Present work
γ	1.161(3)	1.1615(20)	1.1596(20)	1.161	1.1604(8)	1.1615(4)
ν	0.588(1)	0.5880(15)	0.5882(11)	0.5883	0.5881(8)	0.5886(3)
η	0.026(14)	0.027(4)	0.0284(25)	0.0311(10)	0.0285(6)	0.0272(3)
η_2	−0.274(10)	−0.2745(35)	——	——	——	−0.2746(7)
ω	0.794(6)	0.800(40)	0.812(16)	0.810	0.803(3)	0.790(6)
g^*	1.421(4)	1.421(8)	1.413(6)	——	——	1.423(3)

The results for ν obtained by summation of different series (in view of relations $\gamma = \nu(2-\eta)$, $\nu^{-1} = 2 + \eta_2 - \eta$, $\nu = (1-\gamma)/\eta_2$) are presented in Table 2. The fourth estimate is rather inaccurate and will be ignored, while the first three estimates practically coincide; the relative shift of the central values for them can be considered as the scale of the non-controllable systematic error,

$$\delta_{syst} \approx 0.0002\,, \tag{12}$$

which appears because the "natural" interpolations for different interdependent functions are not completely consistent. For the two-dimensional case [8], this effect is the main source of the error: a similar estimate gives $\delta_{syst} \approx 0.05$, which is larger than the natural summation error for most functions. At the present case δ_{syst} is rather small.

T a b l e 2.

Different estmates for the exponent ν

Series	Interval for ν	Central value
$\nu^{-1}(g)$	$0.5883 \div 0.5889$	0.5886
$\gamma^{-1}(g)$, $\eta(g)$	$0.5885 \div 0.5891$	0.5888
$\eta_2(g)$, $\eta(g)$	$0.5884 \div 0.5892$	0.5888
$\gamma^{-1}(g)$, $\eta_2(g)$	$0.5848 \div 0.5916$	0.5882

4. The Ising universality class ($n = 1$)

Initial information is given by expansions [3, 5]

$$\beta(g) = -g + g^2 - 0.4224965707g^3 + 0.3510695978g^4 - 0.3765268283g^5 + 0.49554751g^6$$
$$-0.749689g^7 + \ldots + ca^N\Gamma(N+b)g^N + \ldots\,,$$
$$\eta(g) = (8/729)g^2 + 0.0009142223g^3 + 0.0017962229g^4 - 0.0006536980g^5 \qquad (13)$$
$$+0.0013878101g^6 - 0.001697694g^7 + \ldots + c'a^N\Gamma(N+b')g^N + \ldots\,,$$
$$\eta_2(g) = -(1/3)g + (2/27)g^2 - 0.0443102531g^3 + 0.0395195688g^4 - 0.0444003474g^5$$
$$+0.0603634414g^6 - 0.09324948g^7 + \ldots + c''a^N\Gamma(N+b)g^N + \ldots\,,$$

with the parameters [6]

$$a = 0.14777422\,, \quad b = b' + 1 = 4.5\,, \quad c = 0.039962\,, \quad c' = 0.0017972\,, \quad c'' = 0.0062991\,. \qquad (14)$$

The situation is qualitatively analogous to the previous case, and we use the same values for the parameter L_0, i.e. $L_0 = 1$ for $\gamma^{-1}(g)$, $L_0 = 2$ for $\beta(g)$, $L_0 = 3$ for other functions. Admissible interpolations correspond to the intervals $-1.0 < \tilde{N} < 1.2$ for $\beta(g)$, $1.6 < \tilde{N} < 2.2$ for $\eta(g)$, $-0.5 < \tilde{N} < 2.1$ for $\nu^{-1}(g)$, $-6.2 < \tilde{N} < 2.6$ for $\eta_2(g)$, $-1.1 < \tilde{N} < 0.95$ for $\gamma^{-1}(g)$, and the appearance of the interpolation curves is visually close to that for a case $n = 0$ (Figs.1,2). The results are presented in Table 3.

T a b l e 3.

Critical exponents for the Ising case ($n = 1$) from the field theory

	BNM [3]	LG–ZJ [4]	G–ZJ [5]	Kl [10]	J–Kl [11]	Present work
γ	1.241(4)	1.2405(15)	1.2396(13)	1.241	1.2403(8)	1.2411(6)
ν	0.630(2)	0.6300(15)	0.6304(13)	0.6305	0.6303(8)	0.6306(5)
η	0.031(11)	0.032(3)	0.0335(25)	0.0347(10)	0.0335(6)	0.0318(3)
η_2	−0.382(5)	−0.3825(30)	——	——	——	−0.3832(8)
ω	0.788(3)	0.790(30)	0.799(11)	0.805	0.792(3)	0.782(5)
g^*	1.4160(15)	1.416(5)	1.411(4)	——	——	1.4185(25)

5. The XY universality class ($n = 2$)

The detailed discussion of this case is given in the paper [9]. For completeness, we present here the final results of this study (Table 4).

T a b l e 4.
Critical exponents for the XY (or helium) case ($n = 2$) from the field theory

	BNM [3]	LG–ZJ [4]	G–ZJ [5]	Kl [10]	J–Kl [11]	Present work
γ	1.316(9)	1.3160(25)	1.3169(20)	1.318	1.3164(8)	1.3172(8)
ν	0.669(3)	0.6695(20)	0.6703(15)	0.6710	0.6704(7)	0.6700(6)
η	0.032(15)	0.033(4)	0.0354(25)	0.0356(10)	0.0349(8)	0.0334(2)
η_2	−0.474(8)	−0.4740(25)	——	——	——-	−0.4746(9)
ω	0.780(10)	0.780(25)	0.789(11)	0.800	0.784(3)	0.778(4)
g^*	1.406(5)	1.406(4)	1.403(3)	——	——	1.408(2)

6. The Heisenberg universality class ($n = 3$)

Initial information is given by expansions [3, 5]

$$\beta(g) = -g + g^2 - 0.3832262015g^3 + 0.2829466813g^4 - 0.27033330g^5 + 0.3125559g^6$$
$$-0.414861g^7 + \ldots + ca^N\Gamma(N+b)g^N + \ldots\,,$$
$$\eta(g) = (40/3267)g^2 + 0.0010200000g^3 + 0.0017919257g^4 - 0.0005040977g^5 \tag{15}$$
$$+0.0010883237g^6 - 0.001111499g^7 + \ldots + c'a^N\Gamma(N+b')g^N + \ldots\,,$$
$$\eta_2(g) = -(5/11)g + (10/121)g^2 - 0.0525519564g^3 + 0.0399640005g^4 - 0.0413219917g^5$$
$$+0.0490929344g^6 - 0.06708630g^7 + \ldots + c''a^N\Gamma(N+b)g^N + \ldots\,,$$

with the parameters [6]

$$a = 0.12090618\,, \quad b = b' + 1 = 5.5\,, \quad c = 0.0059609\,, \quad c' = 0.0003656\,, \quad c'' = 0.0012813\,. \tag{16}$$

The same values for L_0, as in previous cases, were used. Admissible interpolations correspond to the intervals $-1.0 < \tilde{N} < 1.6$ for $\beta(g)$, $1.6 < \tilde{N} < 2.3$ for $\eta(g)$ $0.4 < \tilde{N} < 2.0$ for $\nu^{-1}(g)$, $-0.6 < \tilde{N} < 2.2$ for $\eta_2(g)$, $0.5 < \tilde{N} < 0.95$ for $\gamma^{-1}(g)$. The results are presented in Table 5.

T a b l e 5.

Critical exponents for the Heisenberg case ($n = 3$) from the field theory

	BNM [3]	LG–ZJ [4]	G–ZJ [5]	Kl [10]	J–Kl [11]	Present work
γ	1.390(10)	1.386(4)	1.3895(50)	1.390	1.3882(10)	1.3876(9)
ν	0.705(5)	0.705(3)	0.7073(35)	0.7075	0.7062(7)	0.7060(7)
η	0.031(22)	0.033(4)	0.0355(25)	0.0350(10)	0.0350(8)	0.0333(3)
η_2	−0.550(12)	−0.5490(35)	——	——	——	−0.5507(12)
ω	0.780(20)	0.780(20)	0.782(13)	0.797	0.783(3)	0.778(4)
g^*	1.392(9)	1.391(4)	1.390(4)	——	——	1.393(2)

7. Discussion

One can see from Tables 1,3,4,5 that there is a good correspondence between the different field theory estimations. A surprisingly good agreement takes place between our results and estimation by Le Guillou – Zinn-Justin [4]: the typical difference of the central values is less than 0.0010, in spite of rather conservative estimation of errors given in [4]. This coincidence is not in any degree incidental: the authors of [4] carried out interpolation of the coefficient function in order to predict one or two of unknown expansion coefficients and used them to give some kind of the expert prediction, but were induced to allow rather large uncertainty of results due to their strong dependence on variation of the summation procedure. On the other hand, recent reevaluation in [5] looks somewhat artificial and has a tendency to shift the results beyond their natural range; in particular, the shift of g^* in comparison with [4] is always made in the direction opposite to ours (Tables 1,3,4,5) [4]. A good agreement can be seen also with variational perturbation theory [10]; it is especially pleasant that taking into account the large order perturbation behavior and more elaborated estimation of errors [11] makes the results more close to ours. A small disagreement still remains for the exponent η but it is on the same level as violation of the relation $\gamma = \nu(2-\eta)$ for the central values of [11].

Now let us discuss the correspondence of our results with other information on the critical exponents, provided by physical experiment, Monte Carlo simulations (MC) and high temperature series (HT) [12].

Case $n = 3$. Overall scattering of the MC and HT results is rather large and in this extent they agree with Table 5. There is a tendency to a small disagreement between our and the most recent MC results ($\gamma = 1.3960(10)$, $\nu = 0.7112(5)$ [13]) but the latter are in the same disagreement with the physical experiments, whose results for γ are grouped around value 1.386 (see Tables 24,25 in [12]). Analogously, the experimental results for the exponent β suggest the mean value 0.365 [12] in the good agreement with our estimate $\beta = 0.3648(4)$ (following from Table 5) and in the worse agreement with value $\beta = 0.3689(3)$ given in [13].

Case $n = 2$. A situation is analogous to the previous case. Overall scattering of the MC and HT results is rather large (see Fig.1 in [9]) but the recent results have a tendency to contradict Table 4 ($\gamma = 1.3178(2)$, $\nu = 0.6717(1)$ $\eta = 0.0381(2)$ [14]). Simultaneously they contradict the experiments in liquid helium, i.e. value

$$\nu = 0.6705 \pm 0.0006\,, \tag{17}$$

obtained by the measurements of superfluid density from the second sound velocity [15], and the results

$$\alpha = -0.01285 \pm 0.00038\,, \qquad \nu = 0.67095(13) \quad [16]\,,$$

$$\alpha = -0.01056 \pm 0.00038\,, \qquad \nu = 0.6702(1) \quad [17]\,, \tag{18}$$

[4] It should be noted that [4, 5] and the present paper use the same information for $\beta(g)$.

$$\alpha = -0.0127 \pm 0.0003\,, \qquad \nu = 0.6709(1) \quad [18]$$

obtained in the satellite measurements of the thermal capacity (the relation $\alpha = 2 - d\nu$ was used).

Case $n = 1$. In this case, the HT and MC results are numerious (see Tables 3,5 in [12]) and can be summarize as

$$\begin{aligned} \gamma &= 1.2372(5) \\ \nu &= 0.6301(4) \\ \eta &= 0.0364(5) \end{aligned} \tag{19}$$

(see Eq.3.2 in [12]). One can see from the Table 3 that beautiful consensus was reached for the exponent ν; on the other hand, values for γ and η in (19) are in the meaningful contradiction with Table 3. The experimental results have large uncertainty and cannot compete with theoretical predictions.

Case $n = 0$. In this case, precise results for the exponent ν can be obtained by direct study of self-avoiding walks on the lattice; due to simplicity of the algorithm, a good statistics can be gathered. The most recent results ($\nu = 0.5876(2)$ [19], $\nu = 0.5874(2)$ [20], $\nu = 0.58758(7)$ [21]) appear in a slight contradiction with our result in Table 1. This contradiction is not very significant and we can avoid it by allowing more wide set of interpolations and extending the error bars; however, somewhat "unnatural" interpolation curves should be used for it. The results for the exponent γ are essentially less precise [12] and cannot compete with Table 1.

We can conclude that the general situation is satisfactory but there is a cause for anxiety related with the exponents γ and η in the Ising case. Disagreement on the scale 0.003 is essentially larger than uncertainty of the recent field theoretical estimations (Table 3) and that of Monte Carlo results. It is also essential that the latter are obtained by different researches and not related with a specific group. At present, the origin of this disagreement is unclear and further investigations are necessary.

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